

# Exponential and Logarithmic Function Exercises

In the following exercises use logarithms and algebra to solve the problem.

1. Say we invest \$10000 in an account that earns 2.3% interest compounded monthly and \$5000 in an account that earns 2.3% interest compounded continuously. When will the two accounts have the same amount of money? When will the continuous interest account have \$1,000,000 more in it than the compounded monthly account.
2. Say we invest \$5000 in an account that earns 2.3% interest compounded continuously. When will the account double in size?
3. Say we invest \$50000 in an account that earns 2.3% interest compounded continuously. When will the account double in size?
4. Say we invest \$1,000,000 in an account that earns 2.3% interest compounded continuously. When will the account double in size?
5. If the population of a small island is growing at a rate of 2.1% with a current population of 100,000 and the island is 200 by 100 miles. How long will it be until each person has only one square foot to stand in?
6. Find the solution(s) to the following exponential equations.
  - (a)  $4^x = 5$
  - (b)  $7^{2x-1} = 5$
  - (c)  $11^x = 12$
  - (d)  $7^{2-x} = 3^{x+1}$
  - (e)  $11^{5-2x} = 7^{3x+1}$
  - (f)  $2^{2-x} = 2^{x+1}$
  - (g)  $e^x = 10$
  - (h)  $10^x = e$
7. Write the following logarithmic expressions as a single logarithm.
  - (a)  $\log_a(x) + \log_a(y) - (\log_a(z) + \log_a(w))$
  - (b)  $5 \log_a(x) + 2 \log_a(y) - \frac{1}{2}(\log_a(z) + \log_a(w))$
  - (c)  $5(\log_a(x) + 2 \log_a(y)) - \frac{1}{2}(\log_a(z) + \log_a(w))$
8. Find the exact value of the following logarithms.
  - (a)  $\log_5(1/125)$
  - (b)  $\log(100000)$
  - (c)  $\log(0.00000001)$
  - (d)  $\ln(e^2 e^3)$
  - (e)  $\log_3(81)$

9. Write the following logarithms as sums and differences of logarithms.

(a)  $\log_5 \left( \frac{x}{yz} \right)$

(b)  $\log_5 \left( \frac{25x}{yz} \right)$

(c)  $\ln \left( \frac{x^5 w^2}{y^2 \sqrt{z}} \right)$

(d)  $\log \left( \frac{1000w^{21}}{\sqrt[5]{y^2 \sqrt{z}}} \right)$

10. Find the solution(s) to the following logarithmic equations.

(a)  $\log_3(2x - 4) = \log_3(x^2 + 9)$

(b)  $\log_3(2x - 4) = \log_3(x^2 - 9)$

(c)  $\ln(x + 5) = 15$

(d)  $\log(2x - 1) = 7$

(e)  $\log(2x - 1) = -7$

(f)  $\ln(x + 5) + \ln(x - 2) = 1$

(g)  $\ln(x + 5) + \ln(x - 2) = 1 + \ln(x)$

(h)  $\log_5(x) + \log_5(2 - x) = \log_5(x + 1)$

(i)  $\log_5(x) - \log_5(2 - x) = 0$

11. Fill in the following charts

(a) Chart 1: Population growth versus Doubling Time.

Population	Growth Rate	Doubling Time
Mexico	1.9% per year	
Japan		346 years
Mozambique	3.3% per year	
Norway	0.5% per year	
Syria		20.4 years
Philippines		31.5 years

(b) Chart 2: Interest Rate versus Doubling Time versus Amount After 5 Years. Assume that the interest is compounded continuously.

Investment	Interest Rate	Doubling Time	Amount After 5 Years
\$35,000	6.2%		
\$5,000			\$7,130.90
	8.4%		\$11,414.71
		11 years	\$17,539.32

(c) Chart 3: Decay Rate versus Half-Life

Substance	Decay Rate	Half-Life
Polonium		3 min.
Lead		22 years
Iodine-131	9.6% per day	
Krypton-85	6.3% per year	
Strontium-90		25 years
Uranium-238		4500 years
Plutonium		23,105 years