Quadratic Functions and Modeling

The following activity is a modification of one I did as a junior in a high school physics class. In physics (at that moment in history) we needed to rely on ruler measurements and stopwatch timing to examine the phenomena. The calculations were not difficult but the measurements were a bit on the tedious side. Today, using just TI graphing calculator technology we can toss our rulers and stopwatches away.

This activity will get many data points of a physical phenomena that is parabolic in nature. Using these data points you can use the computer to dynamically model the function or use the quadratic regression feature of the graphing calculator to obtain the "best" quadratic through the data.

The idea is to have a ramp of some kind and a ball. If we roll the ball up the ramp (by giving it one push at the bottom) the ball will roll up the ramp, stop and then roll back down. If we could measure the distance from the top of the ramp to the ball at every moment the distance verses time graph would be a parabola. This is because the ball would have a constant acceleration on it due to gravity. The trick is to do all of these measurements. We need to measure time and distance at many points while a ball rolls up and down a ramp. By hand it would be impossible to do this with any accuracy. To illustrate that point take the stopwatch built into your digital Timex and try to stop it exactly at one minute. So to get the measurements we will use a TI-83 and a CBR (Ranger).

Here are the steps for the experiment.

1. Setup a ramp and attach the ranger to the ramp with the clamp that is provided with the ranger.
2. Have the person who is going to push the ball up the ramp practice the push. They need to get the ball up to a point the is about $1\frac{1}{2}$ feet from the ranger. If you get closer than this the ranger does not measure well and if you are too far away the graph will not have enough points on it.
3. Connect the calculator to the ranger.
4. Select Apps $\succ$ CBL/CBR. Press Enter to move onto the next screen.
5. Select Ranger. Press Enter to move onto the next screen.
6. Select Setup/Sample.
7. Make sure that you have the following options on the setup page
   (a) Real Time: Yes
   (b) Time: 10 to 15
   (c) Display: Dist
   (d) Begin On: Enter
   (e) Smoothing: None
   (f) Units: Feet
8. Select Start Now.

9. Press Enter to begin data collection.
10. Very shortly after the CBR begins to flash have the pusher push the ball up the ramp.
11. Let the CBR finish and transfer the data to the calculator. You now have parabolic data and a bunch of garbage outside of that.
12. Press Enter to go back to the menu and select the Select Domain option.
13. Mark off the region that defines the parabolic data and press Enter to select the new domain.
14. At this point you should have a nice parabola on the screen, opening upward.
15. There is only one problem and that is the time is off. Your graph starts at some time (probably close to $\frac{1}{2}$ or 1 second. We would like to have started the ball at the precise moment the ranger started but this is not easy to do and really should not be tried. Instead we will shift the time back so that it starts at 0. To do this type in the following to the calculator.
    (a) 2^{nd} L_1
    (b) $-$
    (c) 2^{nd} L_1
    (d) (1)
    (e) Sto▷
    (f) 2^{nd} L_3
    (g) Enter

    This will put the times starting at zero in the L_3 list.
16. Now using the StatPlot, graph L_1 verses L_3.
17. For a computer modeling experiment take several of the data points and plot them into either GSP or WinGeom. Also put in a general quadratic with dynamic coefficients. Now manipulate the coefficients so that the quadratic passes through the points and note the coefficient values.
18. For a calculator quadratic regression select Stat ≻ Calc ≻ QuadReg. Give it the lists L_3 and L_1 and note the coefficients that are produced. If you do both exercises the coefficients should be close.

You can use this for a number of different courses at different levels. For example, just graphing the data gives you a nice parabola that you can find the equation to in a number of different ways. You can further discuss the free-fall equation $d(t) = \frac{1}{2}at^2 + v_0t + d_0$ and using the coefficients of the equation discuss what the acceleration of the ball down the ramp, the initial velocity that the ball was pushed up the ramp and the initial distance from the ball to the top of the ramp.