Week 12 Activities

These exercises are to establish the relationship between the non-horizontal asymptote and long division. From page 125 of the text you know that

$$\frac{f(x)}{g(x)} = Q(x) + \frac{R(x)}{g(x)}$$

where $Q(x)$ and $R(x)$ are the quotient and remainder when you long divide $g(x)$ into $f(x)$. That is, they are the polynomials on top of the division sign and at the bottom of the subtraction list. Also from the discussion on pages 125 – 131 you know that the function $Q(x)$ is the asymptotic function. So if we relate these two ideas we see that we can find the asymptotes by the algebraic method of long division and we can do the long division by graphically experimenting with the asymptote. That is, experimentally fitting the asymptote to the curve to determine $Q(x)$.

1. Use algebra to determine the asymptote.
   (a) Do long division to divide $g(x) = x^2 + 2x - 5$ into $f(x) = 3x^3 + 5x^2 - 15x + 8$. Call the quotient $Q(x)$.
   (b) Graph the functions

   $$h(x) = \frac{3x^3 + 5x^2 - 15x + 8}{x^2 + 2x - 5} \qquad \textit{and} \qquad Q(x)$$

   on the same graph. Make sure that the x range is big enough so you see $Q(x)$ getting really close to $h(x)$.
   (c) What is the degree of $Q(x)$ and why?
2. Use algebra to determine the asymptote.
   (a) Do long division to divide $g(x) = x^2 + 2x - 5$ into $f(x) = x^4 + x^3 - 8x^2 + 5x + 8$. Call the quotient $Q(x)$.
   (b) Graph the functions

   $$h(x) = \frac{x^4 + x^3 - 8x^2 + 5x + 8}{x^2 + 2x - 5} \qquad \textit{and} \qquad Q(x)$$

   on the same graph. Make sure that the x range is big enough so you see $Q(x)$ getting really close to $h(x)$.
   (c) What is the degree of $Q(x)$ and why?

3. Use the asymptote to determine the quotient.

   (a) If we do the following division

   $$\frac{x^3 - 4x^2 + 11x - 2}{x^2 - 3x + 7}$$

   (b) What will the degree of $Q(x)$ be and why?

   (c) Graph $h(x) = \frac{x^3-4x^2+11x-2}{x^2-3x+7}$ along with a general polynomial having the same degree as $Q(x)$. That is, if you have determined that $Q(x)$ has degree one then you should graph $ax + b$, if you have determined that $Q(x)$ has degree two then you should graph $ax^2 + bx + c$, and so on.

   (d) Use the sliders to "fit" $Q(x)$ as an asymptote to the function.

   (e) Do the division $\frac{x^3-4x^2+11x-2}{x^2-3x+7}$ to verify that your asymptote is actually $Q(x)$.

4. Use the asymptote to determine the quotient.

   (a) If we do the following division

   $$\frac{x^4 - 3x^3 + 9x^2 - 6x + 16}{x^2 - 3x + 7}$$

   (b) What will the degree of $Q(x)$ be and why?

   (c) Graph $h(x) = \frac{x^4-3x^3+9x^2-6x+16}{x^2-3x+7}$ along with a general polynomial having the same degree as $Q(x)$. That is, if you have determined that $Q(x)$ has degree one then you should graph $ax + b$, if you have determined that $Q(x)$ has degree two then you should graph $ax^2 + bx + c$, and so on.

   (d) Use the sliders to "fit" $Q(x)$ as an asymptote to the function.

   (e) Do the division $\frac{x^4-3x^3+9x^2-6x+16}{x^2-3x+7}$ to verify that your asymptote is actually $Q(x)$.