Week 13 Activities

These exercises are aimed at examining logistic growth. Logistic growth is a type of exponential growth that is better suited for modeling the growth of things that have some type of restriction to their maximum size. For example, many biological organisms experience this type of growth since most will not grow indefinitely. You don't tend to see 40 foot humans or house cats that are three stories tall. Populations of organisms also exhibit this type of growth. Most populations are restricted by their environment. A school of fish in a lake have only so much space to occupy. Even humans, who have a tendency to change their environment, are restricted by the Earth. The logistic growth equation is given by

$$f(t) = \frac{a}{1 + be^{-kt}}$$

where a, b and k are constants, t is the independent variable denoting time and $f(t)$ would represent the size of the thing (organism or population) being modeled.

1. Using WinPlot or GSP graph the logistic growth equation using a, b and k as parameters. That is, setup sliders for them. Manipulate the parameters a, b and k and describe what each parameter "controls" and how the graph changes when the parameter is changed. What is the general shape of the graph? What are some of the attributes about graphs we have studied that apply to this graph? When a graph increases more rapidly then its rate of increase is large and when a graph increases less rapidly then its rate of increase is small. If a curve goes from increasing more rapidly to increasing less rapidly the point where it changes is called the point of inflection. What can you say about the rate of increase for this graph?

2. The following set of data comes from an experiment by Reed and Holland in 1919. The data is the average heights (in cm) of a sample of sunflowers during the first 12 weeks of growth. You could easily do this as an experiment in some of your classes, possibly using slightly smaller plants.

Days	Height
0	0.00
7	17.93
14	36.36
21	67.76
28	98.10
35	131.00
42	169.50
49	205.50
56	228.30
63	247.10
70	250.50
77	253.80
84	254.50

   (a) Graph the points on the same worksheet as the one you used for the first activity.

   (b) Manipulate the parameters a, b and k so that the logistic equation best fits the data. Write down the a, b and k numbers you obtained.

   (c) Using a TI-83 or the Voyage 200 do a logistic regression of the data. Note that the Voyage 200 has two different logistic regressions, a logistic and a logistic83. The logistic83 fits a logistic curve of the form

$$f(t) = \frac{a}{1 + be^{-kt}}$$

to the data and the logistic fits a logistic curve of the form

$$f(t) = \frac{a}{1 + be^{-kt}} + d$$

to the data. Also note that you will want to exclude the data values of 0 days and 0 height since these will produce an error. That is, start at the 7 days line. What are the values of a, b and k that were produced by the calculator?

(d) On either the computer or the calculator graph your equation with the calculator's equation and the data. Relate the differences between your equation and the calculator's equation to the physical attributes of the sunflower.

(e) Using the graphs give a short description of the overall growth of the sunflower. Is there a point of inflection? If so, where?

(f) Use the calculator to find the linear, quadratic and cubic regression lines for the data. Graph these with the data. Do any of these better represent the growth of the sunflower? Why or why not?

3. The following set of data comes from the world human population. The dates before 2002 are estimates based on population counts and those after 2002 are estimated based on growth rates. This data is a portion of the total midyear population estimates from the US Census. You can find the entire table for the dates 1950–2050 at http://www.census.gov/ipc/www/worldpop.html.

Year	Population
1950	2,555,360,972
1960	3,039,669,330
1970	3,708,067,105
1980	4,454,607,332
1990	5,275,407,789
2000	6,078,684,329
2010	6,812,009,338
2020	7,515,218,898
2030	8,127,277,506
2040	8.646,671,023
2050	9,078,850,714

(a) Plot the data from 1950 to 2000 in WinPlot or GSP. Also put in a general logistic curve with sliders. To keep the data values more manageable I would replace 1950 with 0, 1960 with 1, 1970 with 2, and so on. I would also do the populations in billions, that is, 2,555,360,972 would be 2.555, 3,039,669,330 would be 3.040, and so on. Fit the data with the curve and write down your a, b and k values.

(b) Use the calculator to fit a logistic curve to the 1950 to 2000 data. Write down the calculator values for a, b and k.

(c) Graph them both on the same axes with the 1950 to 2000 data. Which seems to be a better fit?

(d) Plot the 2010 to 2050 data on the same axes. Which estimate seems to be better, yours or the calculators?

(e) Redo the calculator fit using all of the data.

(f) By this model, what is the maximum human population of the Earth? Is there a point of inflection? If so, where and what does it mean with respect to the human population of the planet?